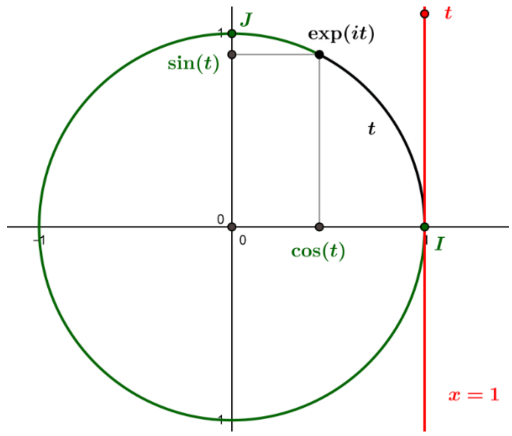
## Exponentielle imaginaire

<table>
<tr><td>Définition exponentielle complexe</td><td>L'ensemble des nombres complexes de module 1 est noté $\mathbb{U}$<br>Soit $z$ un tel complexe, alors il existe $t$ réel tel que $z = \cos t + i \sin t$.<br>On pose la notation $\cos t + i\sin t = e^{it}$<br>$e^{it}$ est appelée l'exponentielle imaginaire.</td><td>
</td></tr>
<tr><td>Remarque</td><td colspan="2">$e^{it} = e^{it'} \Leftrightarrow t = t'[2\pi]$</td></tr>
<tr><td>Propriétés</td><td colspan="2">$e^{it} * e^{it'} = e^{i(t+t')}$ | $e^{-it} = \overline{e^{it}} = \frac{1}{e^{it}}$ | $\cos t = \frac{e^t+e^{-t}}{2}$ ; $\sin t = \frac{e^t-e^{-t}}{2i}$ ; Formules d'Euler.</td></tr>
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## Preuves

$$e^{it} = \cos t + i\sin t \ ; \ e^{it'} = \cos t' + i\sin t'$$
$$e^{it} * e^{it'} = (\cos t + i\sin t)(\cos t' + i\sin t') = \cos t\cos t' - \sin t\sin t' + i(\cos t\sin t' + \sin t\sin t')$$
$$e^{it} * e^{it'} = \cos(t+t') + i\sin(t+t') = e^{i(t+t')}$$

$$e^{-it} = \cos(-t) + i\sin(-t) = \cos t - i\sin t = \overline{\cos t + i\sin t} = \overline{e^{it}}$$
$$\frac{1}{e^{it}} = \frac{1}{\cos t + i\sin t} = \frac{\cos t - i\sin t}{(\cos t + i\sin t)(\cos t - i\sin t)} = \frac{\cos t - i\sin t}{\cos^2 t + \sin^2 t} = \cos t - i\sin t = \overline{e^{it}}$$

$$\frac{e^t + e^{-t}}{2} = \frac{\cos t + i\sin t + \cos(-t) + i\sin(-t)}{2} = \frac{2\cos t}{2} = \cos t$$
$$\frac{e^t - e^{-t}}{2i} = \frac{\cos t + i\sin t - (\cos(-t) + i\sin(-t))}{2} = \frac{2i\sin t}{2i} = \sin t$$

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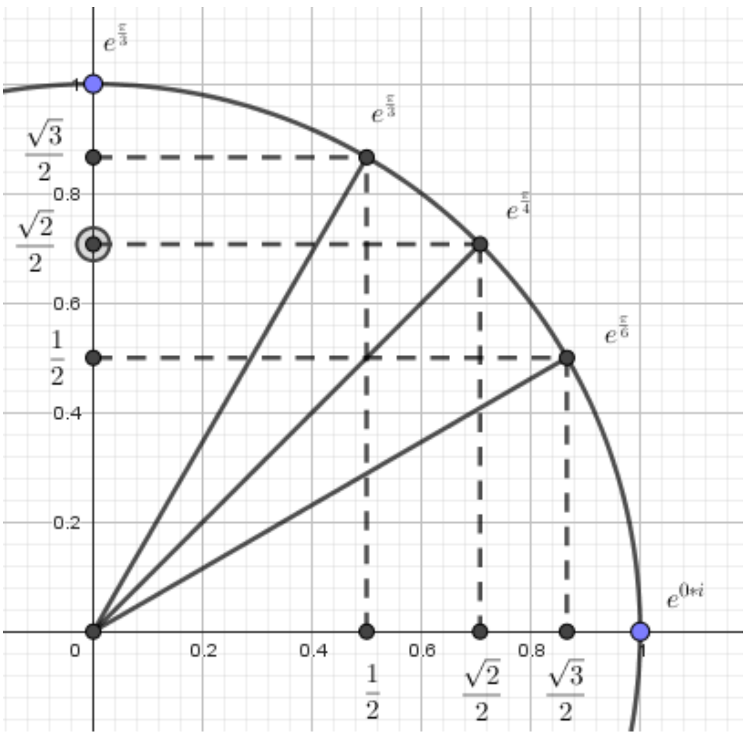
|---|---|
| **Propriétés** | Formule de Moivre : $(\cos x + i\sin x)^n = \cos nx + i\sin nx$ |
| **Preuve** | $(\cos x + i\sin x)^n = (e^{ix})^n = e^{inx} = \cos nx + i\sin nx$ |
| **Exemples** |  |