Nombres complexes. Cours.

Technique angle moitié (Noyau de dirichlet)		
Propriété	$1+e^{it} = 2\cos\left(\frac{t}{2}\right) e^{\frac{it}{2}}$	$1-e^{it} = 2\cos\left(\frac{t}{2}\right) e^{\frac{it}{2}}$
Preuve	$1+e^{it} = e^{\frac{it}{2}}\left(e^{-\frac{it}{2}}+e^{\frac{it}{2}}\right) = e^{\frac{it}{2}} * 2\cos(\frac{t}{2})$	$1-e^{it} = e^{\frac{it}{2}}\left(e^{-\frac{it}{2}}-e^{\frac{it}{2}}\right)$ $= e^{\frac{it}{2}} * -2i\sin\left(\frac{t}{2}\right) = -2isin(\frac{t}{2})\, e^{\frac{it}{2}}$
Propriété	$e^{ip}+e^{ip} = 2e^{ip}\cos\left(\frac{q-p}{2}\right)e^{\frac{i(q-p)}{2}}$	$e^{ip}-e^{ip} = -2ie^{ip}\sin\left(\frac{q-p}{2}\right)e^{\frac{i(q-p)}{2}}$
Preuve	$e^{ip}+e^{iq} = e^{ip}\left(1+e^{i(q-p)}\right) = 2e^{ip}\cos\left(\frac{q-p}{2}\right)e^{\frac{i(q-p)}{2}}$	$e^{ip}-e^{iq} = e^{ip}\left(1-e^{i(q-p)}\right)$ $= -2ie^{ip}\sin\left(\frac{q-p}{2}\right)e^{\frac{i(q-p)}{2}}$
Propriété	$\sum_{k=0}^{n}\cos(kt) = \begin{cases} n+1 \ si\ t\ est\ multiple\ de\ 2\pi \\ \frac{\sin\left(\frac{t(n+1)}{2}\right)}{\sin\left(\frac{t}{2}\right)}\cos\left(\frac{tn}{2}\right)\ sinon \end{cases}$	$\sum_{k=0}^{n}\sin(kt) = \begin{cases} 0 \ si\ t\ est\ multiple\ de\ 2\pi \\ \frac{\sin\left(\frac{t(n+1)}{2}\right)}{\sin\left(\frac{t}{2}\right)}\sin\left(\frac{tn}{2}\right)\ sinon \end{cases}$

Preuve

$$\sum_{k=0}^{n}\cos(kt) = \sum_{k=0}^{n}\Re(e^{ikt}) = \Re\left(\sum_{k=0}^{n}e^{ikt}\right)$$

- Si $t = 2\pi * n\ (n \in \mathbb{N})$

$$\sum_{k=0}^{n}\cos(kt) = \Re(\mathrm{n}+1) = \mathrm{n}+1$$

- Si $t \neq 2\pi * n\ (n \in \mathbb{N})$

$$\sum_{k=0}^{n}\cos(kt) = \Re\left(\frac{1-e^{i(n+1)t}}{1-e^{it}}\right) = \Re\left(\frac{-2isin\left(\frac{t(n+1)}{2}\right)e^{\frac{i(n+1)t}{2}}}{-2isin\left(\frac{t}{2}\right)e^{\frac{it}{2}}}\right) = \Re\left(\frac{\sin\left(\frac{t(n+1)}{2}\right)e^{\frac{int}{2}}}{\sin\left(\frac{t}{2}\right)}\right)$$

$$\sum_{k=0}^{n}\cos(kt) = \Re\left(\frac{\sin\left(\frac{t(n+1)}{2}\right)}{\sin\left(\frac{t}{2}\right)}\left(\cos\left(\frac{tn}{2}\right)+isin\left(\frac{tn}{2}\right)\right)\right)$$

$$\sum_{k=0}^{n}\cos(kt) = \frac{\sin\left(\frac{t(n+1)}{2}\right)}{\sin\left(\frac{t}{2}\right)}\cos\left(\frac{tn}{2}\right)$$

De même

$$\sum_{k=0}^{n}\sin(kt) = \sum_{k=0}^{n}\mathcal{I}(e^{ikt}) = \mathcal{I}\left(\sum_{k=0}^{n}e^{ikt}\right)$$

- Si $t = 2\pi * n\ (n \in \mathbb{N})$

$$\sum_{k=0}^{n}\sin(kt) = \mathcal{I}(\mathrm{n}+1) = 0$$

- Si $t \neq 2\pi * n\ (n \in \mathbb{N})$

$$\sum_{k=0}^{n}\sin(kt) = \frac{\sin\left(\frac{t(n+1)}{2}\right)}{\sin\left(\frac{t}{2}\right)}\sin\left(\frac{tn}{2}\right)$$